Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MM04

Unit Mechanics 4

Wednesday 24 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 Three forces, acting in the x–y plane, act at the points with coordinates as listed below, where a and b are constants and **i** and **j** are unit vectors parallel to the x-axis and y-axis respectively.

Force	Acting at point
$2a\mathbf{i} + \mathbf{j}$	$(1, 3)$
$8b\mathbf{i} - 2a\mathbf{j}$	$(-1, 4)$
$11\mathbf{i} + 4b\mathbf{j}$	$(-5, -2)$

These three forces form a couple.

(a) Find the value of a and the value of b.

[3 marks]

(b) Determine the magnitude of the couple.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

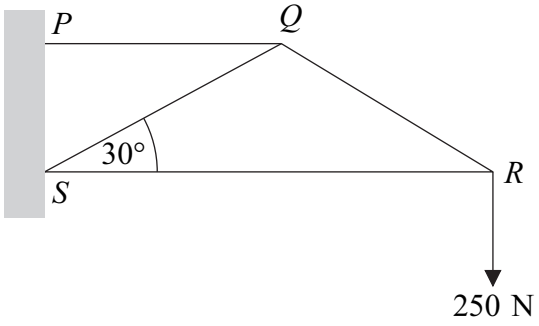
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2 A wall-mounted sign is modelled by a framework $PQRS$. The framework consists of four light smoothly jointed rods PQ, QR, RS and QS. It is smoothly hinged to the wall at P and S. The rods QR and QS have length $2\,\text{m}$ and angle QSR is $30°$. The rods PQ and RS are horizontal. The rods are all in the same vertical plane which is perpendicular to the wall. A weight of $250\,\text{N}$ hangs from R. The framework, as shown in the diagram, is in equilibrium.



(a) Find the magnitude of the force in rod QR, stating whether this rod is in tension or compression.

[3 marks]

(b) Determine the magnitude of the force that the wall exerts on the framework at S.

[5 marks]

QUESTION PART REFERENCE

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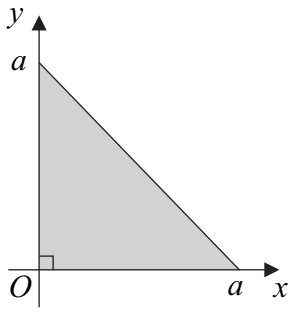
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3 A uniform triangular lamina is bounded by the line $y = a - x$ and the positive x and y axes, as shown in the diagram.



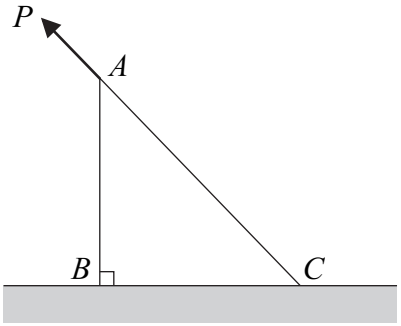
(a) (i) Explain why the centre of mass of the lamina lies on the line $y = x$.

[1 mark]

(ii) By using integration, find the coordinates of the centre of mass of the lamina.

[5 marks]

(b) A lamina is in the shape of an isosceles right-angled triangle, ABC, with its equal sides AB and BC of length a. The lamina has weight W. It rests in a vertical plane with the side BC on a rough horizontal surface. The coefficient of friction between the lamina and the surface is μ. A force of magnitude P acts at A in the direction parallel to CA, as shown in the diagram. The magnitude of the force is gradually increased.



(i) Show that when the lamina is on the point of toppling about B, $P = \dfrac{W\sqrt{2}}{3}$.

[3 marks]

(ii) The lamina slides before it topples if and only if $\mu < k$. Determine the value of k.

[6 marks]

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4 The forces $3\mathbf{i} - 2\mathbf{j}$, $4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{j} - 4\mathbf{k}$ act at the points with coordinates $(-1, 1, 0)$, $(2, 0, 5)$ and $(-6, 2, 1)$ respectively.

(a) Find the total moment of these forces about the origin.

[5 marks]

(b) This system of forces is equivalent to a single force $\mathbf{F}$.

(i) Find $\mathbf{F}$.

[1 mark]

(ii) Determine a vector equation, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, for the line of action of $\mathbf{F}$.

[6 marks]

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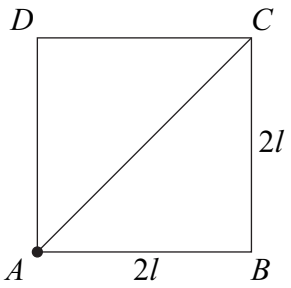
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5 A rigid square framework is formed from five uniform rods, AB, BC, CD, AD and AC. Rods AB, BC, CD and AD are identical with length $2l$ and mass m. Rod AC has mass $2m$.



The framework can rotate freely in a vertical plane about a horizontal axis through A, perpendicular to the plane of the square $ABCD$.

(a) Find the moment of inertia of the rod AC about the axis through A.

[2 marks]

(b) Find the moment of inertia of the rod BC about the axis through A.

[2 marks]

(c) Show that the moment of inertia of the whole framework about the axis through A is $\frac{56}{3}ml^2$.

[3 marks]

(d) The framework is released from rest with AB horizontal and D vertically above A. Find the angular velocity of the framework, in terms of g and l, when B is vertically below A.

[5 marks]

(e) At the instant when B is vertically below A, rod AB collides with a stationary particle, P, of mass $3m$, where $AP = \frac{1}{3}l$ and P is vertically below A. Throughout the subsequent motion, P sticks to the rod AB. Find, in terms of g and l, the angular velocity of the framework immediately after impact.

[5 marks]

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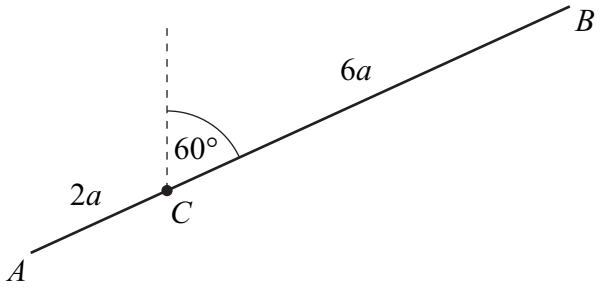
6 A uniform rod AB has length $8a$ and mass m. The point C lies on the rod such that $AC = 2a$. The rod is free to rotate in a vertical plane about a smooth fixed horizontal axis through C which is perpendicular to the rod.

(a) Prove by integration that the moment of inertia of the rod about this axis is $\frac{28}{3}ma^2$.

[4 marks]

(b) Initially the rod is held at rest in the position making an angle of $60°$ with the vertical line through C, as shown in the diagram.

Initial position



The rod is released from rest and begins to rotate. During the subsequent motion the rod makes an angle θ with the upward vertical.

(i) Find the angular acceleration of the rod in terms of a, g and θ.

[3 marks]

(ii) Find the angular velocity of the rod in terms of a, g and θ.

[6 marks]

(iii) The component of the reaction force parallel to the rod at C is X. Find X in terms of m, g and θ.

[3 marks]

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END OF QUESTIONS

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